從社會選擇與阿羅不可能定理看人工智慧的發展

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簡介

在社會選擇理論中, 美國史丹福大學 (Stanford University) 肯尼斯·阿羅 (Kenneth Arrow) 教授於 1951 年發表重要的定理:

> If we exclude the possibility of interpersonal comparisons of utility, then the only methods of passing from individual tastes to social preferences which will be satisfactory and which will be defined for a wide range of sets of individual orderings are either imposed or dictatorial.

若排除人際效用的可比性, 而且在一個相當廣的範圍內, 對任何個人偏好排序集合都有定義, 那麼把個人偏好總合爲社會偏好的理想方法, 要不是強加的, 就是獨裁的.

Kenneth Joseph Arrow

Kenneth Joseph Arrow (1921 – 2017) was an American economist, mathematician, writer, and political theorist. He was the joint winner of the Nobel Memorial Prize in Economic Sciences with John Hicks in 1972.

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阿羅不可能定理

有 m 個決策者, 要從 n 種方案中, 選擇最佳方案. 每位決策者都對 n 個選擇有一個從優至劣的排序. 他們希望設計一種法則 (Algorithm), 將這 m 個排序匯總成一個群體決策. 這種法則需滿足以下條件：

1. 一致性 (unanimity)
   即如果所有的決策者都認爲選擇 a 優於 b, 那麼在群體決策中, a 也必須優於 b.

2. 獨立於無關選項 (independence of irrelevant alternatives)
   如果現在一些決策者改了主意, 但是在每個決策者的排序中, a 和 b 的相對位置不變, 那麼在群體決策中 a 和 b 的相對位置也不變.

3. 非獨裁 (non-dictatorship)
   不存在一個決策者 X, 使得群體決策總是等同於 X 的排序.

如果可選擇的方案數 $n \geq 3$, 我們不可能設計出這種制度.

社會選擇

1. 公民選舉
2. 學生選學校就讀
3. 公司選擇最佳發展方案

可比較

1. 自然數 $\mathbf{N}$

   $1 < 2 < 3 < \cdots$

2. 整數 $\mathbf{Z}$

   $\cdots - 3 < -2 < -1 < 0 < 1 < 2 < 3 < \cdots$

3. 有理數 $\mathbf{Q}$

   $\frac{1}{3} < \frac{1}{2}, \quad \frac{2}{5} < \frac{4}{7}, \ \cdots$

4. 實數 $\mathbf{R}$

   $e < \pi, \ \cdots$

可比較

1. 金牌 > 銀牌 > 銅牌 > ...

2. 甲等 > 乙等 > 丙等 > ...

3. 初級 < 中級 < 高級 < ...

甲組, 乙組, 丙組 ?
鳳梨, 橘子, 蘋果 ?

不可比較

1. 複數 $\mathbb{C}$
   $2+3i : 3+2i$?

2. 社會選擇
   大學排名: 師資, 設備, 國際化, 研究成果, $\cdots$

多面向的指標: (v_1, v_2, v_3, v_4)
$(80, 90, 80, 70) < (90, 80, 70, 80)$?

策略: 各校發展自己的特色, 學生可依自己的喜好選擇適合他的學校.

QS 大學排名

QS 亞洲大學排名的指標中, 學術聲譽占30%, 雇主聲譽占10%, 師生比例占20%, 平均論文被引用率占15%, 教師論文平均發表量占15%, 國際與國內學生占比, 各占2.5%, 來校與赴他校的交換學生占比, 各占2.5%.

QS 世界大學排名的特色, 相較於其他大學排名, 更著重於學校的「學術聲譽」以及「國際化的程度」.

實例

有 7 個委員, 要從 3 個方案中, 選擇最佳方案

1 : $A > B > C$

2 : $A > C > B$

2 : $B > C > A$

2 : $C > B > A$

每人投 1 票, 最高票勝出:

A	3	V
B	2	
C	2	

每人投 1 票, 最多票者先淘汰:

A	4	X
B	2	
C	1	

實例

1 : $A > B > C$

2 : $A > C > B$

2 : $B > C > A$

2 : $C > B > A$

每人投 2 票, 最高票者勝出:

A	3	
B	5	
C	6	V

B 出局了 (少蓋一個章):

A	3	
B	0	
C	4	V

其他方案

評分法: 給每個選項 0 至 $n-1$ 分, 最高分者勝出.

Allan Gibbard (1973), and Mark Satterthwaite (1975)
Any deterministic ordinal electoral system that choose a single winner, one of the following three things must hold:

1. The rule is dictatorial: there exists a distinguished voter who can choose the winner;
2. The rule limits the possible outcomes to two alternatives only; or
3. The rule is susceptible to tactical voting: in certain conditions some voter's sincere ballot may not defend their opinion best.

Example

3 位決策者, 爲 4 個方案評分:

3	2	1	0
A	B	C	D
C	B	D	A
C	B	D	A

A	B	C	D
3	6	7	2
		V	

3	2	1	0
B	A	D	C
C	B	D	A
C	B	D	A

A	B	C	D
2	7	6	3
	V		

註: 評分法並不符合獨立於無關選項之要求.

Implications

G. A. Hazelrigg (1996)
Implications of Arrow's Impossibility Theorem on Approaches to Optimal Engineering Design. *Journal of Mechanical Design*, 118(2), 161-164.

> Pursuing the objective of design optimization as defined by the customers via contemporary approaches can lead the designer to highly inappropriate and undesirable designs. As a consequence of this, it becomes apparent that the methods of Total Quality Management (TQM) and Quality Function Deployment (QFD) can lead to highly erroneous results.

Implications

Jon Kleinberg (2002)
An Impossibility Theorem for Clustering. MIT Press.

> It has been very difficult to develop a unified framework for reasoning about clustering at a technical level. We suggest a formal perspective on the difficulty in finding such a unification, in the form of an impossibility theorem: for a set of three simple properties, we show that there is no clustering function satisfying all three. Relaxations of these properties expose some of the interesting (and unavoidable) trade-offs at work in well-studied clustering techniques such as single-linkage, k-means, and k-median.

Electoral Systems

Political electoral systems:

1. Plurality systems
2. Majoritarian systems
3. Proportional systems
4. Mixed systems

Electoral Systems

Other electoral systems:

1. Condorcet method
2. Ranked pairs
3. Schulze method
4. Borda count

Condorcet Winner

兩兩相比:

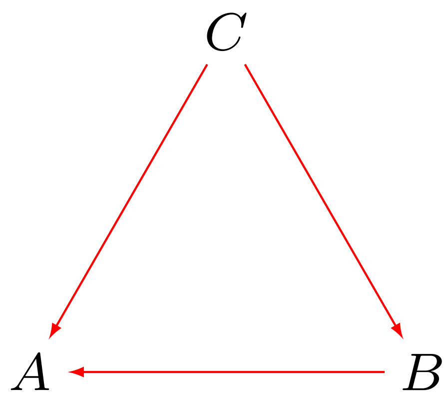
$1 : A > B > C$

$2 : A > C > B$

$2 : B > C > A$

$2 : C > B > A$

$A > B$	3	$B > A$	4
$B > C$	3	$C > B$	4
$C > A$	4	$A > C$	3



人工智慧與大數據分析

1. The birth of AI: Dartmouth Conference 1956
2. The golden years 1956–1974
   (a) 10 年之內電腦將打敗棋王
   (b) 10 年之內電腦可證明新的數學定理
   (c) 20 年之內電腦可以從事任何人類的工作
3. The first AI winter 1974–1980
   (a) 電腦計算能力有限
   (b) Computational Complexity

4. Boom 1980–1987
   (a) 1972: 醫療專家系統 MYCIN
   (b) 1980: 電腦客製化訂製 XCON
   (c) 1981: 日本第五代電腦計劃
   (d) 1982: John Hopfield 證明 neural network 可以學習
5. The second AI winter 1987–1993
   (a) AI 硬體 (LISP 電腦) 昂貴, PC 崛起
   (b) XCON 維護困難且費用昂貴
   (c) 日本第五代電腦計劃未能達到預期目標

6. AI 1993–2011

   (a) 電腦硬體快速進步 Moore's law

   (b) 智慧代理人 Intelligent agents

   (c) 充分使用已有的數學工具

7. AI 2011 – Present

   (a) Deep Learning

   (b) Big Data

   (c) 1997 IBM Deep Blue chess machine

   (d) 2017 Google AlphaGo, AlphaGo zero

困難處

2018/8/11: 2011 年諾貝爾經濟學獎獲得者 Thomas J. Sargent

> 人工智能其實就是統計學, 只不過用了一個很華麗的辭藻.

用舊知識可以創造出新東西嗎 ?

2018/10/07: 某知名教授

我玩了 30 多年 AI, 也發表了一些論文. 基本上, 有標準答案, 且有方法可以找答案的, 因爲機器變快了, 所以機器可以解決 (如: 下棋). 沒有標準答案而需要智慧的, 還是問神比較準.

有可能期望 AI 比神準嗎 ?

人工智慧的機會與挑戰

Assume general algorithms for Solving the Problem do not exist.

Methods for Solving a specific instance (or a specific class of instances) can also be very useful.

Gordon Tullock. 1967.
The General Irrelevance of the General Impossibility Theorem
The Quarterly Journal of Economics, 81(2), 256–270.

Approximation

In many cases, solving a hard problem approximately can also be useful.

Example: Block Chain Consensus

Consensus problem in distributed system

> In asynchronous network environment, consensus is impossible.

How Bitcoin and Ethereum work ?

Conclusions

Theory and Practice

1. Mathematics and Physics

2. Mathematics and Artificial Intelligence ?

Domain knowledge plays a very important role in solving problems effectively.

Thank you.